

**Parvatibai Chowgule College of Arts and Science**  
**Autonomous**  
B.Sc. Semester End Examination, January 2022

**Semester : III**

**Subject: Mathematics**

**Title: Differential Equations I (SEC)**

**Duration : 2 hours**

**Max Marks : 60**

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**Instructions:**

- All Questions are compulsory. However, internal choice is applicable.
  - Figures to the right indicate full marks.
  - Justify all responses.
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1. **Attempt ANY THREE of the following:** **( 3 x 4= 12)**

- (a) Verify whether  $e^{\frac{x}{2}}$  and  $xe^{\frac{x}{2}}$  are solutions of the differential equation  $4y'' - 4y' + y = 0$ . Hence write the general solution.
- (b) Solve the differential equation  $(y + x + 5)\frac{dy}{dx} = (y - x + 1)$
- (c) Verify whether the given differential equation is exact or not. If exact solve the equation.  
 $(e^y + \cos(y)\cos(x))dx = (\sin(y)\sin(x) - xe^y)dy$ .
- (d) Solve the differential equation  $xy' + y = x^2y^2$
- (e) Solve  $x(\sin\frac{y}{x})\frac{dy}{dx} = y(\sin\frac{y}{x}) + x$ .

2. **(A) Attempt ANY ONE of the following:** **(6)**

- (a) If  $(1 + x)$  is one solution of the differential equation  $(x^2 - x)y'' + (x + 1)y' - y = 0$ . Then find its other solution and hence write the general solution.
- (b) Explain the method of variation of parameters to find the particular solution of the non homogeneous linear differential equation.

**(B)** Solve the initial value problem  $y'' + y = 0$ ,  $y(\frac{\pi}{4}) = \frac{1}{2\sqrt{2}}$  and  $y'(\frac{\pi}{4}) = \sqrt{2}$ . **(6)**

3. **Answer the following questions.** **(6 + 6 = 12)**

- (a) Find the general solution of the differential equation  $y'' - 8y' + 15y = 9xe^{2x}$ ;  $y(0) = 5$  and  $y'(0) = 10$ , by undetermined coefficients.
- (b) Consider the forced vibration in the underdamped case and find the frequency for which the amplitude  $\frac{F_0}{\sqrt{(k - \omega^2 M)^2 + \omega^2 c^2}}$  attains the maximum

P.T.O.

4. Answer the following questions.

(6 + 6 = 12)

- (a) Use the wronskian to prove that the two solutions of the homogeneous equation  $y'' + P(x)y' + Q(x)y = R(x)$ , on the interval  $[a, b]$  are linearly dependent if
- they have a common zero  $x_0$  in the interval
  - they have maxima or minima at the same point  $x_0$  in the interval.
- (b) Solve the differential equation  $x''(t) + (\frac{g}{l})x(t) = (\frac{g}{l})L$  using inverse D operator . where  $g, l, L$  are constants subject to the condition  $x(t) = a$  at  $t = 0$  and  $\frac{dx}{dt} = 0$  at  $t = 0$ .

5. Answer ANY TWO of the following.

(2 x 6 = 12)

- (a) Solve  $(2x^2y^2 + y)dx + (3x - x^3y)dy = 0$  using integrating factor.
- (b) Find the general solution of  $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin(2x)$ .
- (c) By making the substitution  $z = \frac{y}{x^n}$  or  $y = zx^n$  and choosing a convenient value of  $n$ , show that the following differential equation can be transformed into equation with separable variable, and hence solve it  $\frac{dy}{dx} = \frac{y-xy^2}{x+x^2y}$ .

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